Biostatistics 140.655

Lab 2

**Topics:**

* Exploratory analysis of within subject correlation within a longitudinal dataset
* Parametric models for within subject correlation
* Weighted least squares
* Comparison of the bias and variance in estimating a linear slope using various modeling assumptions about the within subject covariance structure.

**Learning Objectives:**

Students who successfully complete this lab will be able to:

* Estimate the sample autocorrelation function
* Estimate the autocorrelation function and describe each element of the autocorrelation function
* Deduce a parametric model for the within subject correlation based on the sample autocorrelation function
* Fit a weighted least squares regression model
* Describe statistical properties of the weighted least squares solution

**Scientific Background**:

Assume you are a researcher interested in mental health symptoms among critically ill ICU survivors. You administered the Short Form (36) Health Survey (SF-36) to 100 patients that consented to participate in your study. The SF-36 will be administered at hospital discharge (time 0) and then monthly for 4 months. You are specifically interested in the mental health score of the SF-36.

*A priori* you believe that the mental health symptoms of the ICU survivors will improve over the course of the follow-up, and you state that you will estimate the improvement in mental health symptoms comparing 1 to 4 months post hospital discharge to hospital discharge (time 0 or baseline).

NOTE: We are going to assume we have no deaths in our study patients or drop-out/missing data. We will address these issues later in the course.

**Lab Exercise:**

1. You will explore the autocorrelation function within three hypothetical studies.

Each of the three hypothetical studies was generated assuming the following:

Your goal is to fill in the “?” within the variance specification for this k-variate normal distribution (k = 5). To fill in the “?’ you will be identifying the parametric model that defines the within subject correlation.

Go to the Courseplus site and find data from the three hypothetical studies: “autocor1.csv”, “autocor2.csv” and “autocor3.csv”.

Fill in the following table:

| Lag | Hypothetical Study 1  Sample  autocorrelation function | Hypothetical Study 2  Sample  autocorrelation function | Hypothetical Study 3  Sample  autocorrelation function |
| --- | --- | --- | --- |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0.856 | 0.825 | 0.779 |
| 2 | 0.820 | 0.836 | 0.586 |
| 3 | 0.741 | 0.788 | 0.383 |
| 4 | 0.731 | 0.819 | 0.269 |
| Parametric model: |  |  |  |

**STATA:**

*regress* y i.time

*predict* resid, resid

autocor resid time id

**R:**

fit <- *gls*(y ~ *as.factor*(time), data)

*ACF*(fit, form= ~ 1|id)

1. For each of the three hypothetical studies, estimate the monthly improvement in SF-36 mental health scores using both ordinary least squares (OLS) and weighted least squares (WLS) with an unstructured variance model; treat time as a linear variable. Fill in the table below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Monthly improvement in SF-36 | | | |
| Data | Coefficient | | SE | |
|  | OLS | WLS | OLS | WLS |
| autocor1 | 2.830 | 2.877 | 0.329 | 0.175 |
| autocor2 | 2.934 | 2.933 | 0.297 | 0.123 |
| autocor3 | 2.823 | 2.825 | 0.284 | 0.274 |

What is the true monthly improvement in SF-36 mental health scores?

Do the estimated monthly improvement in SF-36 mental health scores differ across the two statistical methods? If so, why?

Do the standard errors for the estimated monthly improvement in SF-36 mental health scores differ across the two statistical methods? If so, why?

**STATA:**

*regress* y time

mixed y time || id: , nocons residuals(un, t(time))

**R:**

fit.ols <- *lm*(y ~ time, data)

fit.wls <- *gls*(y ~ time,

correlation = corSymm(form = ~ 1 | id),

weights=varIdent(form= ~ 1 | *as.factor*(time)),

data=data, method="ML")

1. Above, you compared the estimated monthly improvement in SF-36 mental health scores generated from the OLS and WLS procedures from a single study of 100 patients. Here, we will explore the repeated sampling behavior of the estimated monthly improvement in SF-36 mental health scores assuming various models for the within subject variance across increasing sample sizes. You will identify important patterns in the behavior of the estimates based on the specified model for the variance. NOTE: We are exploring the properties of WLS within the context of no missing data; we will consider missing data in more detail later in the course.

SIMULATION STUDY: Please DO NOT try to run this simulation study on your laptop; I ran the simulation in R on the Department of Biostatistics computing cluster using 25 parallel processing cores and it took a couple of hours to complete.

I generated 10,000 simulated studies each for *m* = 10, 25, 100, 500 and 1000 patients. The patients were sampled from a population of patients whose data follows the k-variate normal distribution (k = 5) below:

Therefore, the patients are sampled from a population where the monthly improvement in the SF-36 mental health scores is 3 units, the variance of the SF-36 mental health scores at any time is 100 and the correlation between any two SF-36 mental health scores is given by and ρ = 0.9; the AR1 model.

In each of the 10,000 simulated studies, I estimated the monthly improvement in SF-36 mental health scores using the correct model for the mean (linear function of month) and the following models for the within subject variance:

* 1. WLS – V known: I provided the correct information for *Vi*; i.e. and
  2. WLS – V estimated: I assumed the correct model for *Vi* but I estimated the required parameters within each of the simulated studies
  3. WLS – V unstructured: I did not assume a model for *Vi*so estimated 5 variance and 10 correlation parameters within each of the simulated studies.
  4. OLS: I assumed the SF-36 mental health scores from the same subject were uncorrelated and that the variance of the SF-36 mental health scores was the same at all the measurement times and estimated the variance within each of the simulated studies.

The table below displays the bias and variance of the 10,000 estimated monthly improvements in SF-36 mental health scores based on different sample sizes and the models i. through iv. The bias is defined as the average of the 10,000 estimated monthly improvements in SF-36 mental health scores over the simulated studies minus 3 (the true monthly improvement).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Sample size (m)** | **Bias** | | | | **Variance** | | | |
| **WLS –**  **V known** | **WLS – V estimated** | **WLS - unstructured** | **OLS** | **WLS –**  **V known** | **WLS – V estimated** | **WLS - unstructured** | **OLS** |
| **10** | -0.005 | -0.005 | -0.007 | -0.003 | 0.422 | 0.422 | 0.678 | 0.442 |
| **25** | -0.004 | -0.004 | -0.003 | -0.003 | 0.172 | 0.172 | 0.200 | 0.181 |
| **100** | -0.002 | -0.002 | -0.002 | -0.002 | 0.0430 | 0.0430 | 0.0443 | 0.0451 |
| **500** | -0.0005 | -0.0005 | -0.0004 | -0.0003 | 0.00883 | 0.00883 | 0.00888 | 0.00925 |
| **1000** | -0.0006 | -0.0007 | -0.0007 | -0.0005 | 0.00439 | 0.00439 | 0.00440 | 0.00458 |

Based on the results in the table answer the following questions:

1. For a fixed sample size, how does the bias compare across the models specified by the within subject variance?
2. Regardless of the model selected for the within subject variance, how does the bias change as the sample size goes from small (m = 10) to large (m = 1000)?
3. Compare the variance across WLS – V known to WLS – V estimated for fixed sample sizes.
4. For small sample sizes (m = 10 and m = 25), compare the variance for WLS – V known and WLS – V estimated to WLS – unstructured? If there are differences, what do you think drives the differences?
5. For larger sample sizes (m = 100 to m = 1000), compare the variance for WLS – V known and WLS – V estimated to WLS – unstructured? If there are differences, what do you think drives the differences?
6. For each of the sample sizes considered, compute the relative efficiency of estimating the monthly improvement assuming the correct variance model to assuming independence.

General properties of WLS and frequently asked questions:

1. Regardless of how we specify the model for V, the WLS procedure produces an unbiased estimate of the monthly improvement in SF-36 mental health scores.
2. However, specifying the wrong model for V can have an impact on your inference! EXAMPLE: the OLS estimator produces an estimate of the variance that is 5% too large!
3. Why not always use the unstructured approach? ANSWER: in small samples, you found that the variance for the WLS- unstructured was inflated and what defines “small samples” will change depending on the complexity of the mean model and n (the number of observations within a subject). In addition, sometimes there are unexpected dependencies in the empirical estimate of V using the unstructured approach so the model doesn’t fit (i.e. we are unable to invert the estimated V).
4. Why not always use OLS? You could if your goal is only estimation (i.e. interest is in prediction) not inference (hypothesis testing, confidence intervals). If you are interested in inference, then you can get the wrong answer!
5. Our simulation study focused on data generated from an underlying multivariate normal distribution. After we estimate the monthly improvement in SF-36 mental health scores, we want to estimate a confidence interval. Our confidence interval methods rely on the assumption that the slope estimate is normally distributed. Even if the data are not normally distributed, the normality of the slope estimates hold in large samples, due to the central limit theorem.